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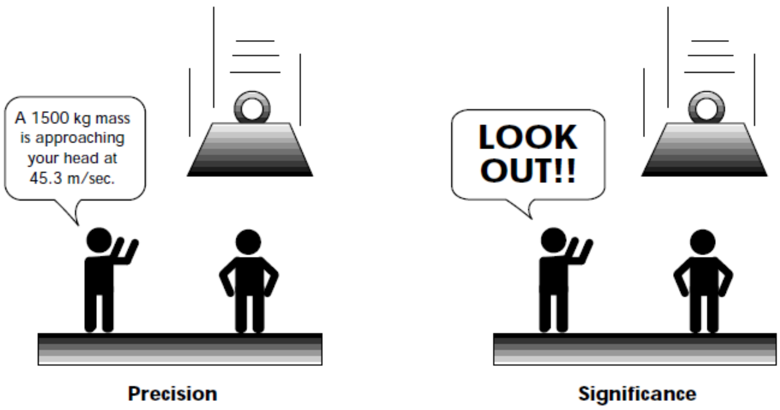
Fuzzy logic: between human reasoning and artificial intelligence



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Abstract

Fuzzy logic is an extension of Boolean logic by Lotfi Zadeh in 1965 based on the mathematical theory of fuzzy sets, which is a generalization of classical set theory. By introducing the concept of degree in the verification of a condition, allowing a condition of being in a state other than true or false, fuzzy logic provides a very valuable flexibility to use reasoning, which makes it possible taking into account the inaccuracies and uncertainties. One of the advantages of fuzzy logic to formalize human reasoning is that the rules are set in natural language.

In this report, we:

- introduce the basic concepts of fuzzy logic,
- propose some arguments which support the view that fuzzy logic can model human reasoning better than standard logic and probability theory,
- conduct an psychological experiment on humans to see if their way of thinking can be reflected by fuzzy logic.

We show that fuzzy logic can explain many experiments that had undermined traditional models of human reasoning in the 20th century. We show how the non-additivity of probability judgments can be expressed in a fuzzy system. We then confront fuzzy logic with some paradoxes of classical logic when it tries to model human reasoning: the sorites paradox is typically the kind of threshold problem that fuzzy logic reduces and the paradox of entailment does not pose a problem in fuzzy logic. It would be interesting to further explore Hempel's paradox and especially how we could express it in a neuro-fuzzy system. Similarly, Wason selection task would require further analysis, this time by focusing on fuzzy modus ponens and modus tollens.

Thus fuzzy logic appears as a powerful theoretical framework for studying human reasoning. Surprisingly, we find only one study comparing the decisions made by humans with that of a fuzzy system, whose purpose was essentially to design a system of decision support for medical personnel, not analyze human reasoning as such. We

conduct our own experiment and investigate whether a fuzzy system could mimic the results observed in humans. For this purpose, we use a technique for optimizing fuzzy system using neural networks (neuro-fuzzy), through which we obtain good results, although the correlation between the two criteria for entry is high: a fuzzy system gives results closer to experimental values than those obtained by a polynomial system. This result reinforces the hypothesis that fuzzy logic can be used to explain decisions from human reasoning.

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Chapter 1

The fuzzy universe

As complexity rises, precise statements lose meaning and meaningful statements lose precision. Albert Einstein.

1.1 The issue

Knowledge available to humans are almost never perfect. These imperfections can be distinguished into two classes:

- **Inaccuracies**, which refer to knowledge whose validity is subject to question. For example, if we know someone bumped his head on a ceiling, we can guess that he is likely to be very tall.
- **Incertitudes**, which refer to knowledge that is not clearly perceived or defined. For example, instead of saying someone is 2 feet and 3 inches, we usually say that person is very tall.

In fact, these imperfections stem from the nature of man and the world: none of our senses and observation instruments allow us to reach an infinite precision and the world is based on the principle of continuity in the mathematical sense, as opposed to discrete values. This is one reason why it is so difficult to establish a system of measurement units and why any quantity is only approximate. Similarly, rare are the situations where we can be totally sure that this statement is true without imposing a prior set of assumptions.

As a result, the knowledge on which human reasoning relies on is almost always marred by a number of uncertainties and inaccuracies. We will not discuss here about scientific reasoning, the objective of which is just to get rid of any imperfections, but about all the usual reasoning that we do every day, constantly, on things, people and thoughts surrounding us. This kind of reasoning ranges from driving a car to the treatment recommended by a doctor to his patient or to the decision to buy a loaf of bread for tomorrow morning.

Surprisingly, and fortunately, despite the vagueness that characterizes our world view, the quality of human reasoning is remarkable. Whether in simple or complex situations, decisions are generally very good compared to the vagueness and uncertainty of the problem's data.

A human being, as part of systems theory, is considered a system in itself [Mélès, 1971], which is able to reason on very imperfect data. His impressive performance, given the complexity and diversity of situations encountered, interested some researchers in systems theory in the 1960s, among whom Lotfi Zadeh, the founder of fuzzy logic.

1.2 Fuzzy logic

Fuzzy logic is an extension of Boolean logic by Lotfi Zadeh in 1965 based on the mathematical theory of fuzzy sets, which is a generalization of the classical set theory. By introducing the notion of degree in the verification of a condition, thus enabling a condition to be in a state other than true or false, fuzzy logic provides a very valuable flexibility for reasoning, which makes it possible to take into account inaccuracies and uncertainties.

One advantage of fuzzy logic in order to formalize human reasoning is that the rules are set in natural language. For example, here are some rules of conduct that a driver follows, assuming that he does not want to lose his driver's licence:

If the light is red...	if my speed is high...	and if the light is close...	then I brake hard.
If the light is red...	if my speed is low...	and if the light is far...	then I maintain my speed.
If the light is orange...	if my speed is average...	and if the light is far...	then I brake gently.
If the light is green...	if my speed is low...	and if the light is close...	then I accelerate.

Intuitively, it thus seems that the input variables like in this example are approximately appreciated by the brain, such as the degree of verification of a condition in fuzzy logic.

1.3 Definitions

To exemplify each definition, we will develop throughout this section a fuzzy inference system the real purpose of which is to determine the tip-up in the end of a meal in a restaurant, based on service quality and the quality of food.

1.3.1 Fuzzy sets

Fuzzy logic is based on fuzzy set theory, which is a generalization of the classical set theory [Zadeh, 1965]. By abuse of language, following the habits of the literature, we will use the terms fuzzy sets instead of fuzzy subsets. The classical sets are also called clear sets, as opposed to vague, and by the same token classical logic is also known as Boolean logic or binary.

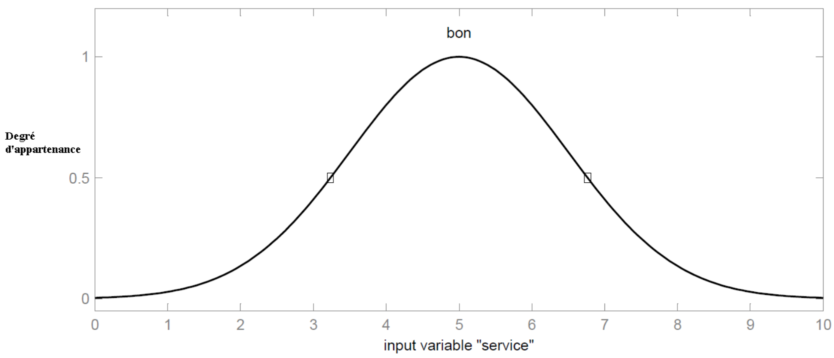


Figure 1.1: Membership function characterizing the subset of 'good' quality of service

The figure 1.1 shows the membership function chosen to characterize the subset of 'good' quality of service.

Definition 1.
*Let X be a set. A fuzzy subset A of X is characterized by a **membership function.** $f^a : X \to [0, 1]$.*
Note: This membership function is equivalent to the identity function of a classical set.

In our tip example, we will redefine membership functions for each fuzzy set of each of our three variables:

- Input 1: quality of service. Subsets: poor, good and excellent.

- Input 2: quality of food. Subsets: awful and delicious.
- Output: tip amount. Subsets: low, medium and high.

The shape of the membership function is chosen arbitrarily by following the advice of the expert or by statistical studies: sigmoid, hyperbolic, tangent, exponential, Gaussian or any other form can be used.

The figure 1.2 shows the difference between a conventional set and a fuzzy set corresponding to a delicious food.

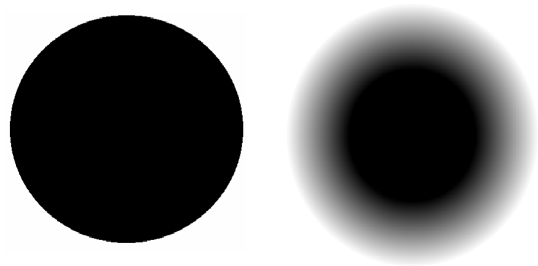


Figure 1.2: Graphical representation of a conventional set and a fuzzy set

The figure 1.3 compare the two membership functions corresponding to the previous set.

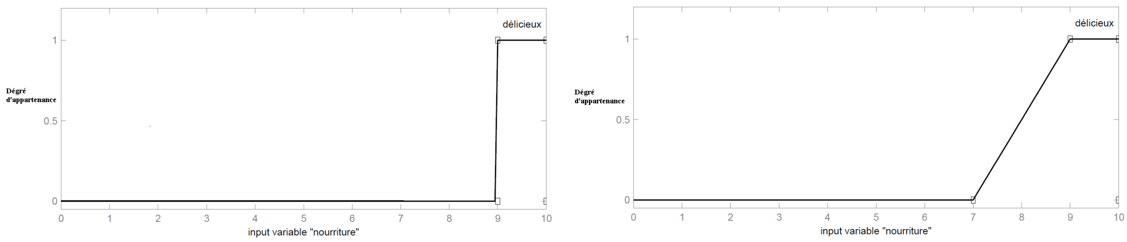


Figure 1.3: Comparison between a identity function of a conventional set and a membership function of fuzzy set

In order to define the characteristics of fuzzy sets, we are redefining and expanding the usual characteristics of classical sets.

Let X be a set and A a fuzzy subset of X and μ_A the membership function characterizing it. $\mu_A(x)$ is called the membership degree of x in A.

Definition 2.
*The **height** of A, denoted $h(A)$, corresponds to the upper bound of the codomain of its membership function: $h(A) = \sup\{\mu_A(x) \mid x \in X\}$.*

Definition 3.
*A is said to be **normalised** if and only if $h(A) = 1$. In practice, it is extremely rare to work on non-normalised fuzzy sets.*

Definition 4.
*The **support** of A is the set of elements of X belonging to at least some A (i.e. the membership degree of x is strictly positive). In other words, the support is the set $\mathrm{supp}(A) = \{x \in X \mid \mu_A(x) > 0\}$.*

Definition 5.
*The **kernel** of A is the set of elements of X belonging entirely to A. In other words, the kernel $noy(A) = \{x \in X \mid \mu_A(x) = 1\}$. By construction, $\mathrm{noy}(A) \subseteq \mathrm{supp}(A)$.*

Definition 6.
*An α-**cut** of A is the classical subset of elements with a membership degree greater than or equal toα : $\alpha\text{-cut}(A) = \{x \in X \mid \mu_A(x) \geqslant \alpha\}$.*

Another membership function for an average tip through which we have included the above properties is presented in Figure 1.4.

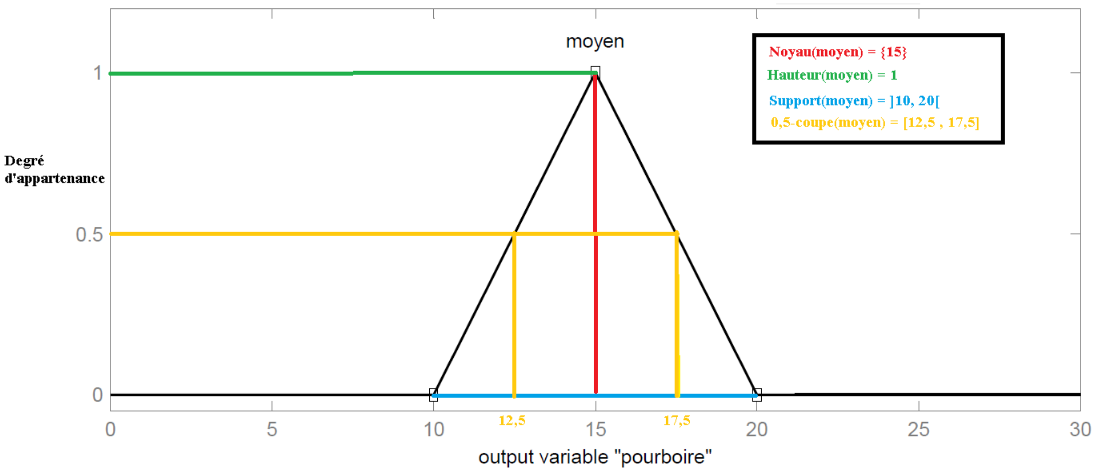


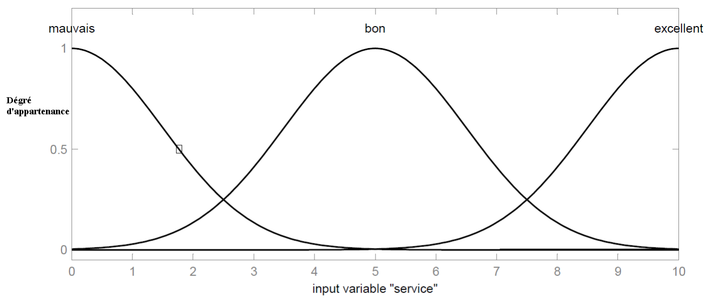
Figure 1.4: A membership function with properties displayed

We note that if A was a conventional set, we would simply have $\mathrm{supp}(A) = noy(A)$ and $h(A) = 1$ (ou $h(A) = 0$ si $A = \emptyset$). Our definitions can therefore recover the usual properties of classical sets. We will not talk about the cardinality property because we will not use this concept later in this report.

1.3.2 The linguistic variables

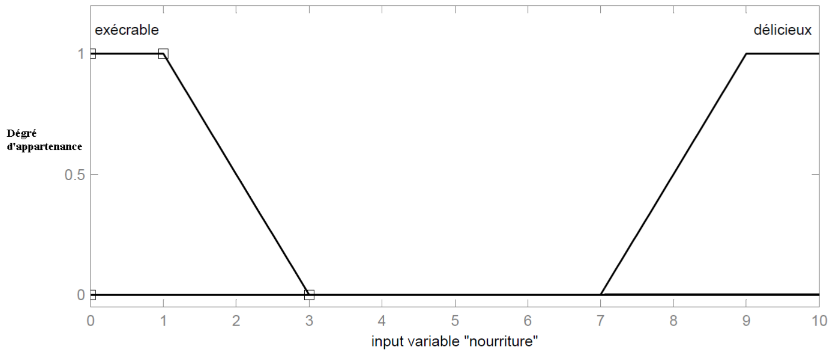
The concept of membership function discussed above allows us to define fuzzy systems in natural language, as the membership function couple fuzzy logic with linguistic variables that we will define now.

Definition 7.
Let V be a variable (quality of service, tip amount, etc.), X the range of values of the variable and T_V a finite or infinite set of fuzzy sets. A ***linguistic variable*** *corresponds to the triplet (V, X, T_V).*



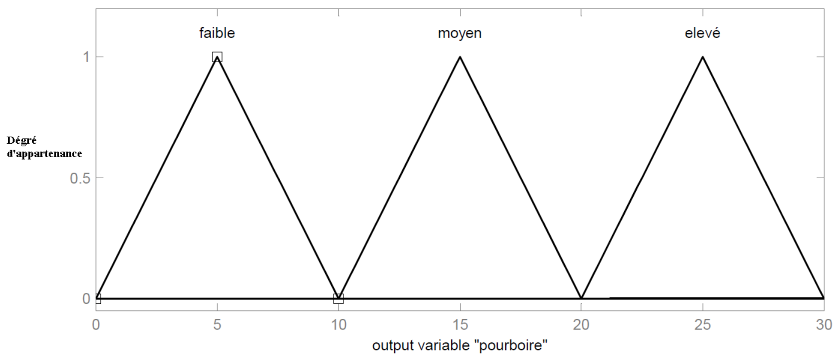
$V = qualité\ du\ service$
$X = \Re^+$
$T_V = \{mauvais, bon, excellent\}$

Figure 1.5: Linguistic variable 'quality of service'



$V = qualité\ de\ la\ nourriture$
$X = \Re^+$
$T_V = \{exécrable, délicieux\}$

Figure 1.6: Linguistic variable 'quality of food'



$V = montant\ du\ pourboire$
$X = \Re^+$
$T_V = \{faible, moyen, élevé\}$

Figure 1.7: Linguistic variable 'tip amount'

1.3.3 The fuzzy operators

In order to easily manipulate fuzzy sets, we are redefining the operators of the classical set theory to fit the specific membership functions of fuzzy logic for values strictly between 0 and 1.

Unlike the definitions of the properties of fuzzy sets that are always the same, the definition of operators on fuzzy sets is chosen, like membership functions. Here are the two sets of operators for the complement (NOT), the intersection (AND) and union (OR) most commonly used:

Name	Intersection AND: $\mu_{A\cap B}(x)$	Union OU: $\mu_{A\cup B}(x)$	Complement NOT: $\mu_{\bar{A}(x)}$
Zadeh Operators MIN/MAX	$min\,(\mu_A(x), \mu_B(x))$	$max\,(\mu_A(x), \mu_B(x))$	$1-\mu_A(x)$
Probabilistic PROD/PROBOR	$\mu_A(x)\times\mu_B(x)$	$\mu_A(x) + \mu_B(x) - \mu_A(x)\times\mu_B(x)$	$1-\mu_A(x)$

With the usual definitions of fuzzy operators, we always find the properties of commutativity, distributivity and associativity classics. However, there are two notable exceptions:

- In fuzzy logic, the law of excluded middle is contradicted: $A\cup\bar{A}\neq X$, i.e. $\mu_{A\cup\bar{A}}(x)\neq 1$.
- In fuzzy logic, an element can belong to A and not A at the same time: $A\cap\bar{A}\neq\emptyset$, i.e. $\mu_{A\cap\bar{A}}(x)\neq 0$. Note that these elements correspond to the set $\mathrm{supp}(A)-\mathrm{noy}(A)$.

1.3.4 Reasoning in fuzzy logic

In classical logic, the arguments are of the form:

$$\begin{cases} Si\ p\ alors\ q \\ p\ vrai\ alors\ q\ vrai \end{cases}$$

In fuzzy logic, fuzzy reasoning, also known as approximate reasoning, is based on **fuzzy rules** that are expressed in natural language using linguistic variables which we have given the definition above. A fuzzy rule has the form:
If $x\in A$ *and* $y\in B$ *then* $z\in C$, with A, B and C fuzzy sets.

For example:
'If (the quality of the food is delicious), then (tip is high)'.

The variable 'tip' belongs to the fuzzy set 'high' to a degree that depends on the degree of validity of the premise, i.e. the membership degree of the variable 'food quality' to the fuzzy set 'delicious '. The underlying idea is that the more propositions in premise are checked, the more the suggested output actions must be applied. To determine the degree of truth of the proposition fuzzy 'tip will be high', we must define the fuzzy implication.

Like other fuzzy operators, there is no single definition of the fuzzy implication: the fuzzy system designer must choose among the wide choice of fuzzy implications already defined, or set it by hand. Here are two definitions of fuzzy implication most commonly used:

Name	Truth value
Mamdani	$min\left(f_a(x), f_b(x)\right)$
Larsen	$f_a(x) \times f_b(x)$

Notably, these two implications do not generalize the classical implication. There are other definitions of fuzzy implication generalizing the classical implication, but are less commonly used.

If we choose the Mamdani implication, here is what we get for the fuzzy rule *'If (the food quality is delicious), then (tip is high)'* where the food quality is rated 8.31 out of 10:

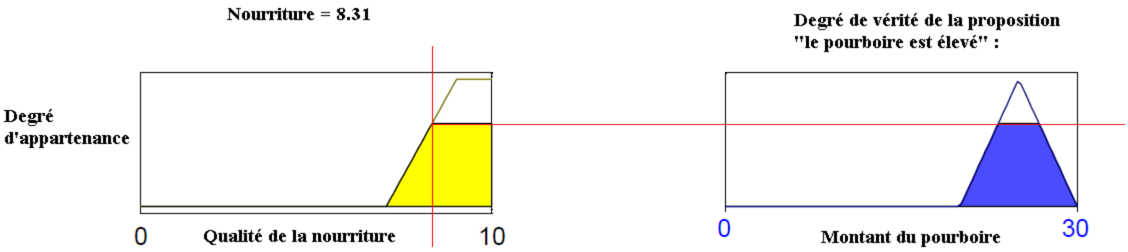


Figure 1.8: Example of fuzzy implication

The result of the application of a fuzzy rule thus depends on three factors:

1. the definition of fuzzy implication chosen,
2. the definition of the membership function of the fuzzy set of the proposition located at the conclusion of the fuzzy rule,
3. the degree of validity of propositions located premise.

As we have defined the fuzzy operators AND, OR and NOT, the premise of a fuzzy rule may well be formed from a combination of fuzzy propositions. All the rules of a fuzzy system is called the **decision matrix**. Here is the decision matrix for our tip example:

If the service is bad or the food is awful	then the tip is low
If the service is good	then the tip is average
If the service is excellent or the food is delicious	then the tip is high

If we choose the Mamdani implication and the translation of OR by MAX, here is what we get for the fuzzy rule *'If (the service is excellent and the food is delicious), then (tip is high)'* where the quality of service is rated 7.83 out of 10 and the quality of food 7.32 out of 10:

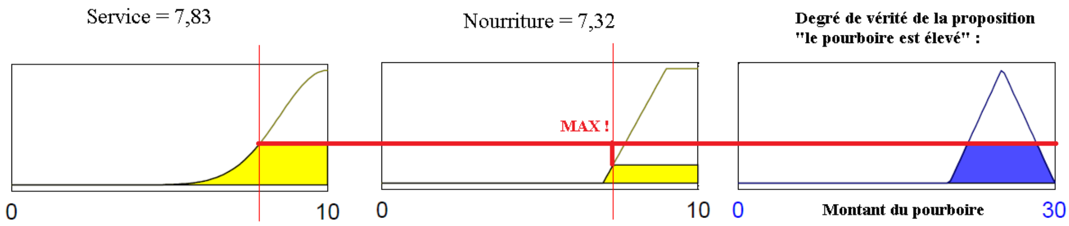


Figure 1.9: Example of fuzzy implication with OR translated by MAX

We will now apply all the 3 rules of our decision matrix. However, we will obtain three fuzzy sets for the tip: we will aggregate them by the operator MAX which is almost always used for aggregation.

As we see, we now has to make the final decision, namely decide how much the tip will be knowing that the quality of service is rated 7.83 out of 10 and quality of food 7.32 out of 10. This final step, which allows to switch from the fuzzy set resulting from the aggregation of results to a single decision, is called the **defuzzification**.

1.3.5 The defuzzification

As with all fuzzy operators, the fuzzy system designer must choose among several possible definitions of defuzzification. A detailed list can be found in the research article [Leekwijck and Kerre, 1999]. We will briefly present the two main methods of defuzzification: the method of the mean of maxima (MeOM) and the method of center of gravity (COG).

The MeOM defuzzification sets the output (decision of the tip amount) as the average

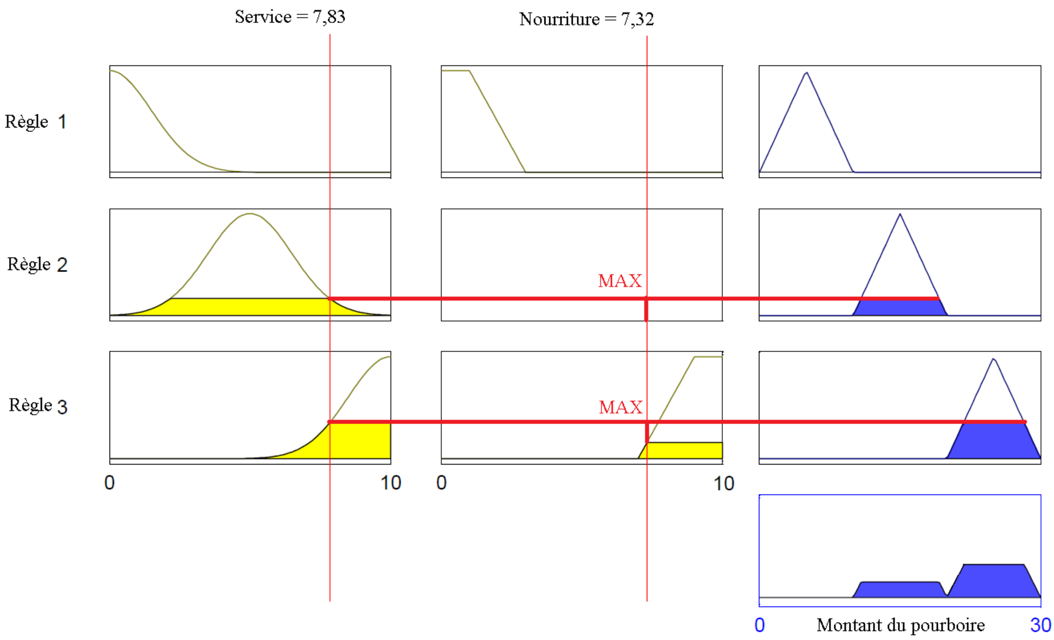


Figure 1.10: Example of fuzzy implication using the decision matrix

of the abscissas of the maxima of the fuzzy set resulting from the aggregation of the implication results.

$Décision = \frac{\int_S y \cdot dy}{\int_S dy}$
where $S = \{y_m \in R, \mu(y_m) = SUP_{y \in R}(\mu(y))\}$
and R is the fuzzy set resulting from the aggregation of the implication results.

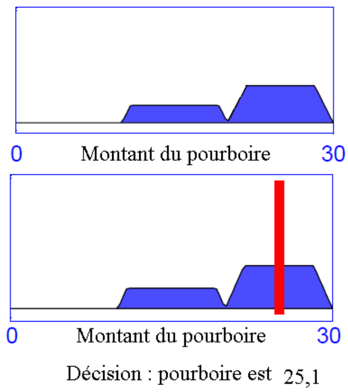


Figure 1.11: Defuzzification with the method of the mean of maxima (MeOM)

The COG defuzzification is more commonly used. It defines the output as corresponding to the abscissa of the center of gravity of the surface of the membership function characterizing the fuzzy set resulting from the aggregation of the implication results.

$$Décision = \frac{\int_S y \cdot \mu(u) \cdot dy}{\int_S \mu(u) \cdot dy}$$

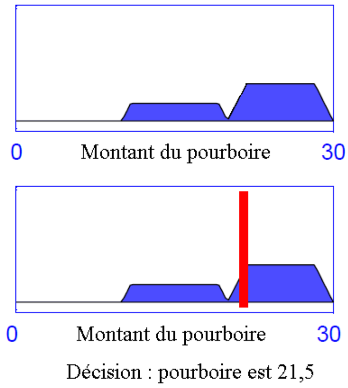


Figure 1.12: Defuzzification with the method of center of gravity (COG)

This definition avoids the discontinuities could appear in the MeOM defuzzification, but is more complex and has a greater computational cost. Some work as [Madau D., 1996] seek to improve performance by searching other methods as effective but with a lower computational complexity. As we see in the two figures showing the MeOM and COG defuzzifications applied to our example, the choice of this method can have a significant effect on the final decision.

1.3.6 Conclusions

In the definitions, we have seen that the designer of a fuzzy system must make a number of important choices. These choices are based mainly on the advice of the expert or statistical analysis of past data, in particular to define the membership functions and the decision matrix.

Here is an overview diagram of a fuzzy system:

In our example,

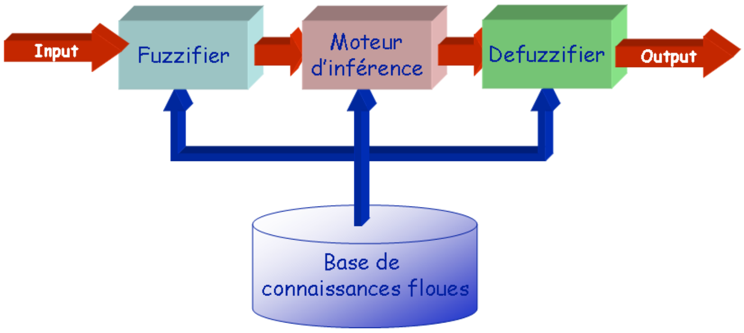


Figure 1.13: Overview diagram of a fuzzy system:

- the **input** is 'the quality of service is rated 7.83 out of 10 and quality of food 7.32 10',
- the **fuzzifier** corresponds to the 3 linguistic variables 'service quality', 'food quality' and 'tip amount',
- the **inference engine** is made of the choice of fuzzy operators,
- the **fuzzy knowledge base** is the set of fuzzy rules,
- the **defuzzifier** is the part where has to be chosen the method of defuzzification,
- the **output** is the final decision: 'the tip amount is 25.1'.

It is interesting to see all the decisions based on each variable with our fuzzy inference system compared to the decisions that we would get using classical logic:

Thus, fuzzy logic allows to build inference systems in which decisions are without discontinuities, flexible and nonlinear, i.e. closer to human behavior than classical logic is. In addition, the rules of the decision matrix are expressed in natural language. We will see in the second chapter if human reasoning in decision-making processes as in the example of the tip has similar results to inferences from fuzzy systems.

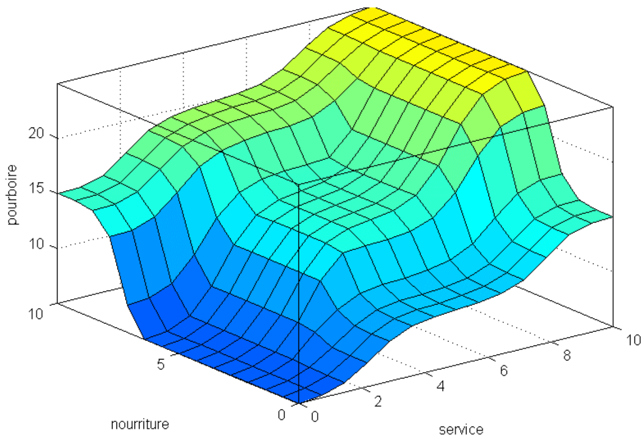


Figure 1.14: Decisions of a system based on fuzzy system

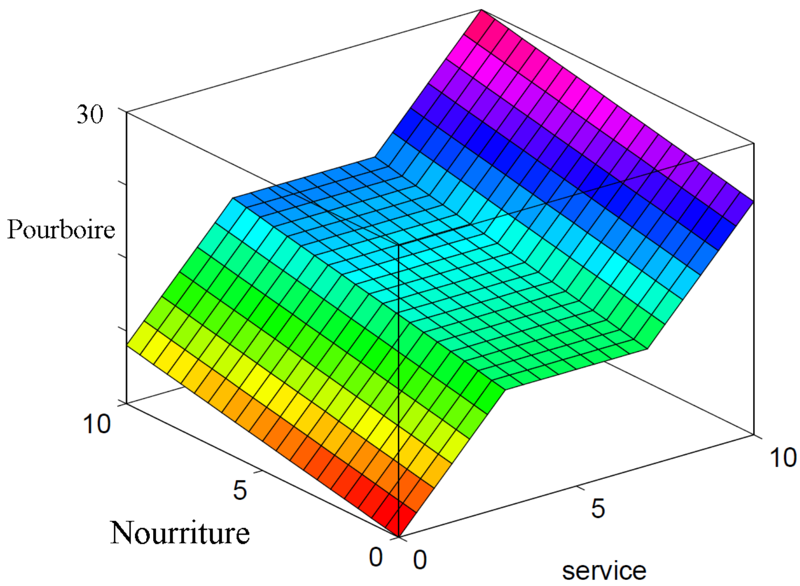


Figure 1.15: Decisions of a system based on classical logic

Chapter 2

Human reasoning

L'être humain va au-delà de l'information immédiatement disponible.
Jerome Bruner.

2.1 The problem

Reason is the ability to think, as it allows human beings to judge well and to apply this judgement to action. It is the ability to develop, from primitive knowledge of the real or hypothetical state of one's environment (**prémisses**), other knowledge or beliefs about the state of the environment (**conclusions**), through activities totally internalized. **Reasoning** means the activity of reason, i.e. the method of passing from premises to conclusion.

The 20^{th} century was a major turning point for the understanding of human reasoning: assumptions that were previously firmly rooted in the common thought were totally challenged, and new theories have emerged to fill the gaps.

However, many experimental observations on human reasoning remain only partially explained: the purpose of this chapter is to see how fuzzy logic can represent a novel approach to the understanding of human reasoning.

To this end, we will address several issues that motivated the development of new theories of human reasoning from the 20^{th} century and whether the fuzzy logic can provide an alternative response.

2.2 Non-additive probabilities

The beginning of probability theory dates back from the correspondence between

Pierre de Fermat and Blaise Pascal in 1654 and was axiomatized by Kolmogorov in 1933 [Shafer G., 2005]. One direct consequence of these axioms is that $P(\Omega \setminus E) = 1 - P(E)$, where Ω is the universe, and E some event.

Probabilities can be interpreted in two ways:

- Objectively: probabilities are values given for events in the context of gambling.
- Subjectively: probabilities correspond to the knowledge of a human being on an event or state of the world.

As the purpose of this report is to study human reasoning, we keep only the second interpretation. In this perspective, the probability value is unique to each human being.

However, it turns out that we can experimentally show that the probabilities used and generated by human reasoning does not follow the Kolmogorov axioms, as they do no respect one of Kolmogorov axioms' direct consequence $P(\Omega \setminus E) = 1 - P(E)$ previously seen.

2.2.1 Subadditivity

[Redelmeier DA, 1995] made the following experiment: the case of a patient admitted to a hospital was summarized and presented to doctors from Stanford. The latters were then asked to estimate the probability of each of the following events:

- The patient died while in hospital,
- The patient will leave alive the hospital but died within one year,
- The patient will die somewhere between 1 and 10 years,
- The patient will live at least 10 more years.

As these four events form a partition of the universe, whatever the life duration of the patient, 1 and only 1 of these events will be true. We would therefore expect that the sum of the probabilities is equal to 1. However, experimentally, we find that the average of the sum of these four probabilities is equal to 1.64 (confidence interval 95% : [1.34, 1.94]). This result shows that the sum of the probabilities of events of a partition of the universe is greater than 1 is called **subadditivity** of probability judgement.

The term subadditivity come from mathematics, which define a subadditive function as follows:

Definition 8.
*A function $f\colon A \to B$ is said to be **subadditive** iff A domain closed under addition, B is a codomain partially ordered closed under addition and $\forall x, y \in A, f(x+y) \leq f(x) + f(y)$.*

2.2.2 Superadditivity

Conversely, some articles such as [Cohen et al., 1956], [Macchi, 1999] and [Boven and Epley, 2003] show that under certain conditions, the probability judgement may reflect a **sur-additivité**, autrement dit que la somme des probabilités des événements d'une partition de l'univers est inférieure à 1.

Definition 9.
*A fontion F is said to be **superadditive** iff it is not subadditive.*

For example, [Macchi, 1999] asked students to evaluate the probability of two events:

- The freezing point of gasoline does not equal that of alcohol. What is the probability that the freezing point of gasoline is greater than that of alcohol?
- The freezing point of gasoline does not equal that of alcohol. What is the probability that the freezing point of alcohol is greater than that of gasoline?

Analysis of the results shows that on average the sum of these probabilities is about 0.9, instead of 1 as we might expect, the first event being the complement of the second. [Macchi, 1999] examines the various factors that may increase or decrease this amount. Broadly, we find that the more the subject is sure of his decision, the more likely he will have subadditive probabilities, and conversely the more uncertain he is about his decision, the more likely he will have superadditive probabilities. The **support theory** [Tversky and Koehler, 1994] predicts in more details the types of non-additivity and provides a unified explanation of the experimental results previously described.

Therefore, human reasoning ignores the axioms of Kolmogorov, who laid the foundations of probability theory. It does not allow to entirely model human reasoning.

2.2.3 Fuzzy logic and non-additivity

In fuzzy logic, as we saw in the first chapter, the law of excluded middle is contradicted ($A \cup \bar{A} \neq X$, i.e. $\mu_{A \cup \bar{A}}(x) \neq 1$) because the definition of the OR is classically $max\,(\mu_A(x), \mu_B(x))$ (Zadeh min/max) or $\mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x)$ (PROD/PROBOR). The problem of non-additivity does not arise. According to the chosen membership functions, we can have what we want: superadditivity, or subadditivity or sum equal to 1.

Let's model our last example on freezing points taken from [Macchi, 1999] in a fuzzy system:

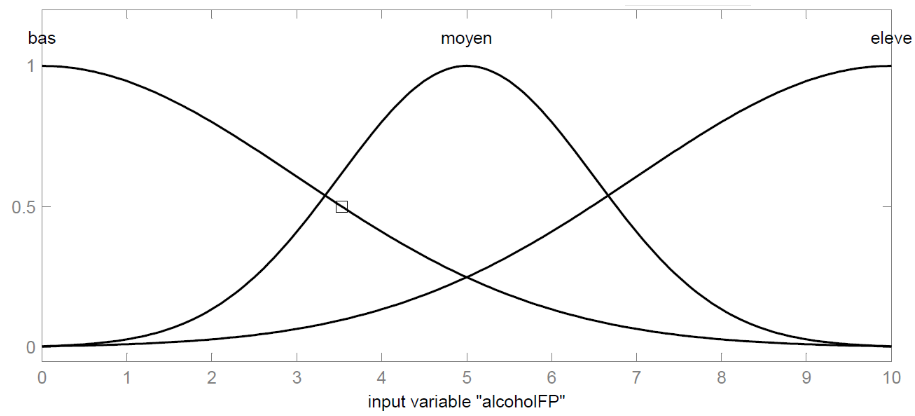


Figure 2.1: Variable freezing point (FP) of alcohol

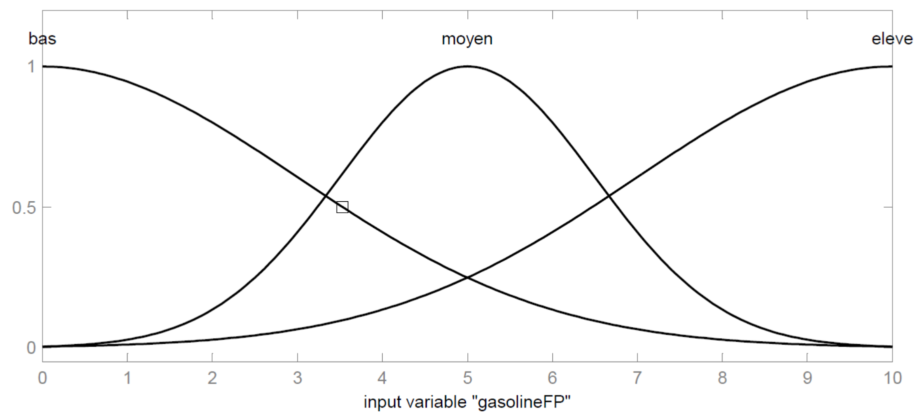


Figure 2.2: Variable freezing point (FP) of gasoline

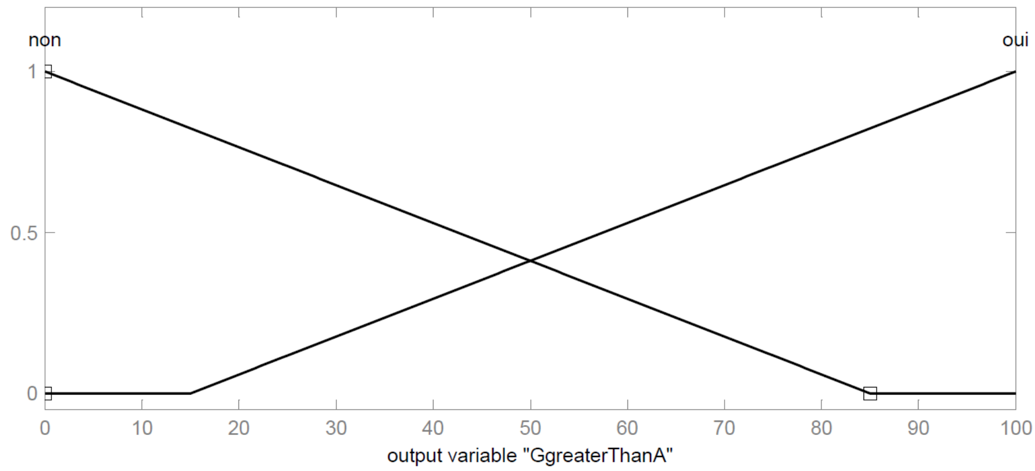


Figure 2.3: Variable freezing point (FP) of gasoline greater than alcohol (GgreaterThanA)

Here the choices of fuzzy operators we do (usual choice):

Operator	Choice
OR	max
AND	min
Implication	Mamdani
Aggregation	max
Defuzzication	COG

The decision matrix will be:

If FP alcohol is low	and if PC gasoline is average	then GgreaterThanA is yes.
If FP alcohol is low	and if PC gasoline is high	then GgreaterThanA is yes.
If FP alcohol is average	and if PC gasoline is low	then GgreaterThanA is no.
If FP alcohol is average	and if PC gasoline is high	then GgreaterThanA is yes.
If FP alcohol is high	and if PC gasoline is low	then GgreaterThanA is no.
If FP alcohol is high	and if PC gasoline is average	then GgreaterThanA is no.

where :

- PC alcohol stands for freezing point of alcohol,
- PC gasoline stands for freezing point of gasoline,
- GgreaterThanA is yes means that the freezing point of gasoline is higher than the freezing point of alcohol,

- GgreaterThanA is no means that the freezing point of gasoline is lower than the freezing point of alcohol.

Imagine that PC alcohol is equal to 5/10 and PC essence is equal to 7.95/10 (as were are not freezing experts, we chose to measure the freezing points on an arbitrary scale of 0 to 10). Our fuzzy system then gives the value GgreaterThanA = 66.7. With this value, we can find the membership degrees in fuzzy sets "yes" and "no":

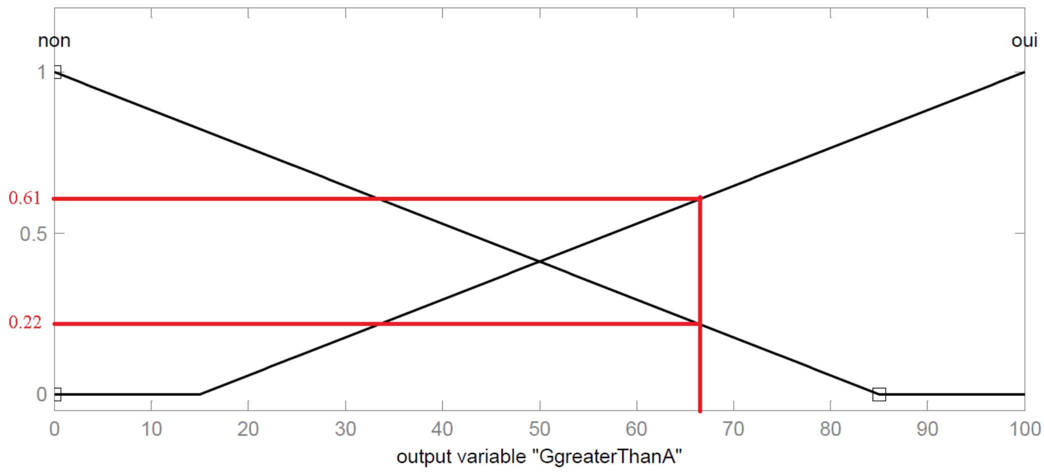


Figure 2.4: Superadditivity obtained by fuzzy logic

The degree of membership in fuzzy set "no" is $\frac{85-66,7}{85} \approx 0,22$ and the degree of membership in the fuzzy set "yes" is $\frac{66,7-15}{85} \approx 0,61$.We find that $0,22 + 0,61 < 1$, which reflects the on-additivity found experimentally in [Macchi, 1999].

More generally, the non-additivity that has been shown in many studies of experimental psychology can be modeled by choosing fuzzy variables and membership functions of their fuzzy sets.

2.3 The cognitive paradoxes of classical logic

Classical logic (binary) often result in conclusions that differ from what humans do in practice. We will see in this section whether fuzzy logic helps to explain some of these differences.

2.3.1 Sorites paradox

The sorites paradoxes arise when we ask questions like "How many grains does it take to do a heap?". The answer is difficult because in everyday language, no added grain can be identified as being the difference between a heap and not a heap. Since one single grain is not regarded as being a heap, it followed that two grains are not a heap, no more than 3, and so on. However, if we continue to add grains, after a number of grains that is not defined, we eventually get a heap.

In classical logic, the proposition "T is a heap" raises problems as it must be either true or false. However, we have just seen that there is no threshold expressed in number of grains above which a non-heap becomes a heap.

In fuzzy logic, this problem is less important because we can define "T is a job" as a fuzzy set: we do not have this "threshold" of classical logic. However, we must still define a membership function.

2.3.2 Paradoxes of material implication

In classical logic, implications are of the type "If A then B". By definition, the implication is always true when A is true. Thus, the implication "if 1 + 1 = 3 then 1 + 1 = 2" is considered true, while cognitively many people do not accept it as true. This is a paradox that stems from the definition of implication.

In fuzzy logic, the system designer chooses all fuzzy rules of the system. Having no interest in putting unnecessary rules like "if 1 + 1 = 3 then 1 + 1 = 2", there will be only useful rules for the purposes of the system.

2.3.3 Hempel's paradox

Consider the statement "All crows are black" (H). This sentence is logically equivalent to "all non-black objects are non-ravens" (H'). Indeed, by the law of contraposition, $P \rightarrow Q$ is equivalent to $\neg Q \rightarrow \neg P$.

As a result, the discovery of a black raven confirms (H) and also (H'), but also that the discovery of something non-black that is not a raven such as a pink flamingo or even a gray umbrella confirms (H') and therefore (H). This conclusion seems paradoxical.

In fuzzy logic, fuzzy sets and decision matrices are determined by the system designer. Nevertheless, it is possible to redefine these sets or fuzzy rules by learning, particularly through neural networks. This combination between fuzzy logic and neural networks is called **neuro-fuzzy** [Jangi, 1992]. We will not go into details here but will do so in the chapter 3.3.4.

2.3.4 Wason selection task

The Wason selection task is an experiment that tests mastery of modus ponens and modus tollens in human subjects [P. C. Wason, 1966]. Here is what is proposed to individuals as described in the original experiment, "Four cards with numerals on one side and a letter on the other, are placed on a table. One side of each card is visible. The visible faces are: D, 7, 5, K. What card(s) do you need to turn over to determine the card(s) that violate(s) the following rule: 'If a card has a D on one side, then it has a 5 on the other side'. Do not return card unnecessarily, nor forget to return one."

Approximately 80% subjects are wrong in their answer. The most common error, namely returning the card 5 and forget the card 7, reveals two cognitive biases:

- verification bias, which is to seek more verification than refutation of the rule,
- matching bias, which is to focus on the items mentioned in the statement.

In logical terms, forgetting the card 7 shows poor control of the modus tollens. Choosing card 5 corresponds to the fallacy of affirming the consequent, that is to say, to confuse a simple relation of implication with a relation of logical equivalence.

Fuzzy logic does not predict the fallacy of affirming the consequent, but can still simulate it without any theoretical problem: the confusion between logical implication and equivalence can not be held unless it is deliberately introduced by the designer system. The same applies to the modus tollens.

Note that a few research groups are currently focusing on the fuzzy modus tollens, including the development of reasoning systems based on experience ([Zhaohao Sun and Sun., 2005]).

2.4 Natural language

Human reasoning mostly rely on statements made in natural language. Classical logic can not adequately maintain the links between the propositions such as:

- "This rabbit is small"

- "Some rabbits are small"
- "Some rabbits are very small"
- "Some rabbits are not very small"

We saw in the first chapter that fuzzy logic is based on the concept of linguistic variables, which allows to introduce considerable flexibility in the characterizations. To better take into account the proposals made in natural language, Zadeh has published three long articles in 1975 to extend the notion of linguistic variables [Zadeh, 1975a] [Zadeh, 1975b] [Zadeh, 1975c], which he resumed in 1978 [Zadeh, 1978] and he laid the foundations of the PRUF theory (*Possibilistic Relational Universal Fuzzy Language*).

In particular, beyond the simple linguistic variables, PRUF introduced four linguistic concepts can be modeled by fuzzy logic:

- modifiers. Example: " Xis very far",
- quantifiers. Example: " Most Chinese have black hair"
- qualifiers. Example: " It is likely that X is far",
- composition operators. Example: " X and Y is near is far." The composition operators correspond to the operators and/or, which we have already seen the first part).

For the sake of brevity, as our point is to show the flexibility of fuzzy logic for the formalization of natural language with respect to classical logic, we will briefly explain how to define modifiers.

Modifiers are adverbs that modify a fuzzy variable in order to increase or diminish its significance as "very", "moderate" or "more". For example, this allows us to obtain the fuzzy set "very dynamic" from the fuzzy set "dynamic" and " very".

Definition 10.
A ***linguistic modifier*** *is an operator m characterized by a function t_m such that if A is a fuzzy set characterized by membership function f_a, $m(A)$ is a fuzzy set characterized by $f_{m(A)}$, with $f_{m(A)} = t_m(f(A))$.*

Some examples of modifiers introduced initially in the articles of Zadeh:

- "'very"' : $t_m(x) = x^2$;

- "'more or less"' : $t_m(x) = \sqrt{x}$;
- "'not"' : $t_m(x) = 1 - x$;

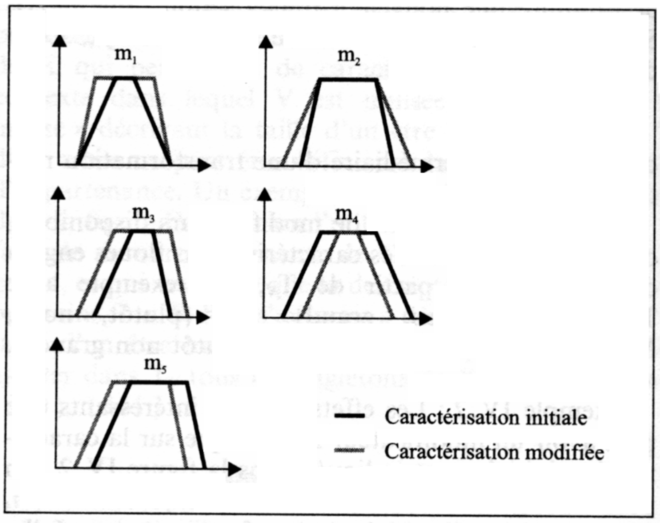


Figure 2.5: Effect of modifiers

Another way to define a modifier is to represent it directly as a fuzzy set [Ribeiro and Moreira, 2003], which allows to handle the same type of elements in the fuzzy system:

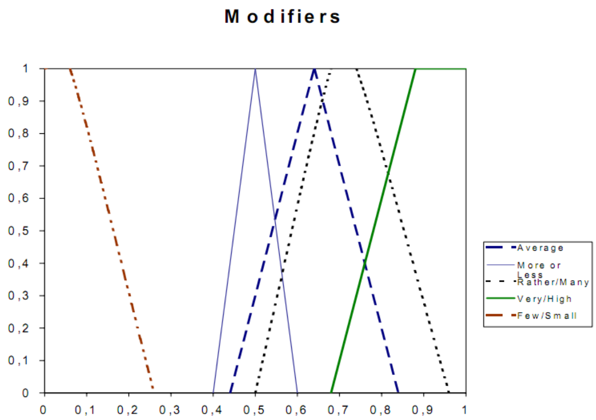


Figure 2.6: Membership functions of modifiers

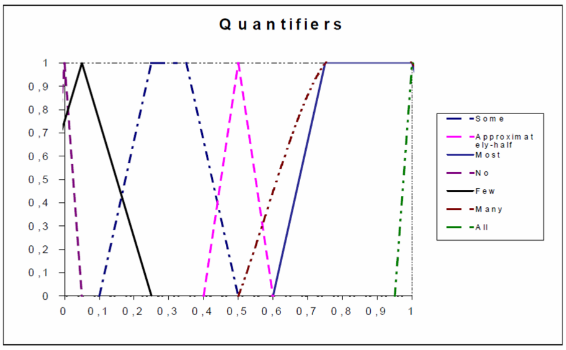


Figure 2.7: Membership functions of quantifiers

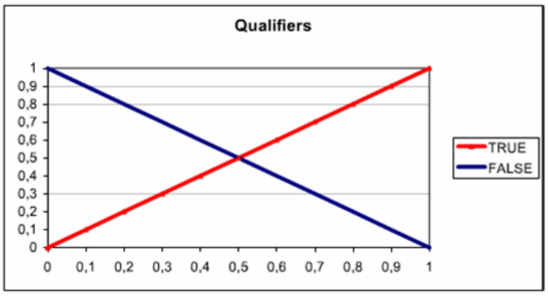


Figure 2.8: Membership functions of qualifiers

2.5 Conclusions

The purpose of this chapter was to assess the "proximity" between human reasoning and fuzzy logic. For this purpose, we studied several aspects of human reasoning that are difficult to model and experimentally undermined classical probabilies and logic.

Fuzzy logic seems to provide a satisfactory answer for the non-additivity of probabilities observed in human subjects. Fuzzy logic can also address or mitigate certain paradoxes that appear in classical logic. Nevertheless, it would be useful to widen the study to better analyze some aspects of these paradoxes in fuzzy logic, including the formalization of the Wason selection task.

An undeniable strength of fuzzy logic is its proximity to the natural language. In essence, the rules of the decision matrix are expressed in natural language, and the basis of fuzzy logic based on fuzzy set theory that lends itself very well to express the linguistic variables.

Fuzzy logic seems to provide a good theoretical framework for human reasoning, at least its imitation. We will see in the next chapter if we can find experimentally traces of fuzzy inference in the decision made by human subjects as part of tasks that will be affected to them.

Chapter 3

Experiments

La vérité n'est pas l'exactitude. Henri Matisse.

3.1 Objectifs

There are few studies comparing the results of inference systems based on fuzzy logic with the results that come from human reasoning. In this chapter we will analyze two aspects of human reasoning through two experiments.

Firstly we will analyze a research paper in the field of medical informatics in which the authors attempted to design a system for decision support based on fuzzy logic to help anesthesiologists during surgery. The prospect will be then to study the type inference in humans when we give them very precise, certain objective variables measured by medical instruments.

In a second step, we will do our own experiment in the world of video games. We will study how gamers give scores to video games based on two criteria they evaluate themselves, contrary to previous experience when evaluating these two criteria is not made by humans but is given by an external source.

3.2 Decision system support for anaesthesiologists

In the study [Hamdi Melih Saraoglu, 2007], the authors have worked on the decision-making of anesthesiologists during surgery. An anesthesiologist decides the content of the gas as the patient breathes continuously: this gas must be more or less anesthetic according to the patient's condition at a given time of the operation.

To make this decision, the anesthesiologist is based on two main criteria (input):

- SAP: systolic arterial pressure (i.e. blood pressure),
- HRP: heart pulse rate.

Based on these two criteria, the anesthesiologist will take a decision (output):

- AO: rates of anesthetics in the gas (anesthesia output).

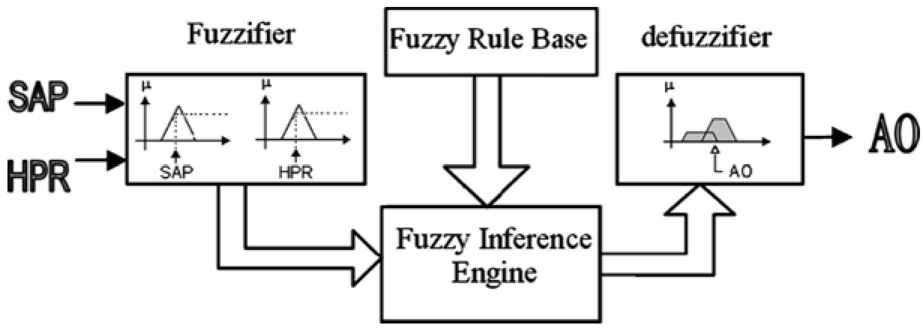


Figure 3.1: Overview of the inference system

Initially, the study authors define the artificial fuzzy system and in a second time they put it in a real situation in order to compare these results with the decisions made by human anesthesiologists. We will first briefly explain the fuzzy system chosen.

First, here are the fuzzy sets and the decision matrix:

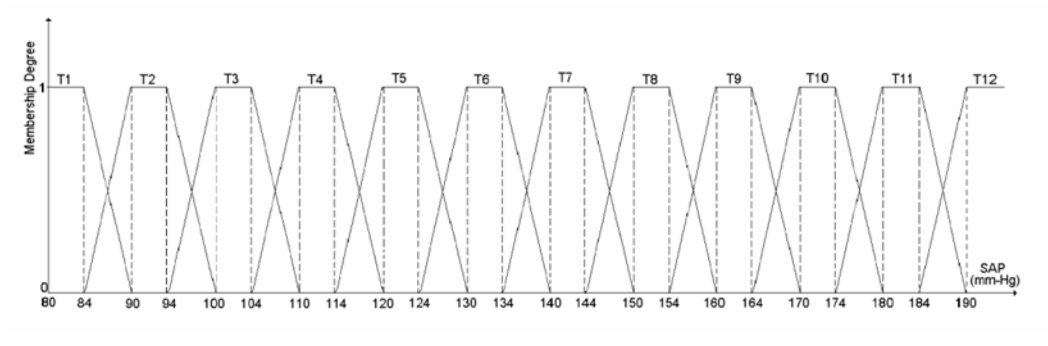


Figure 3.2: Fuzzy sets of the variable SAP

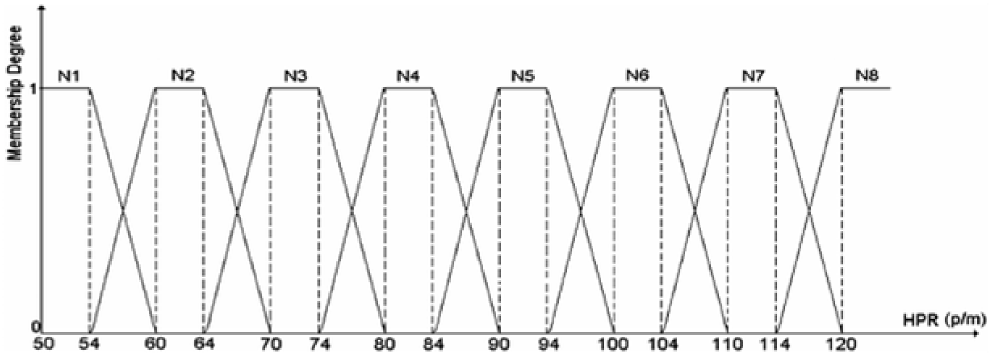


Figure 3.3: Fuzzy sets of the variable HPR

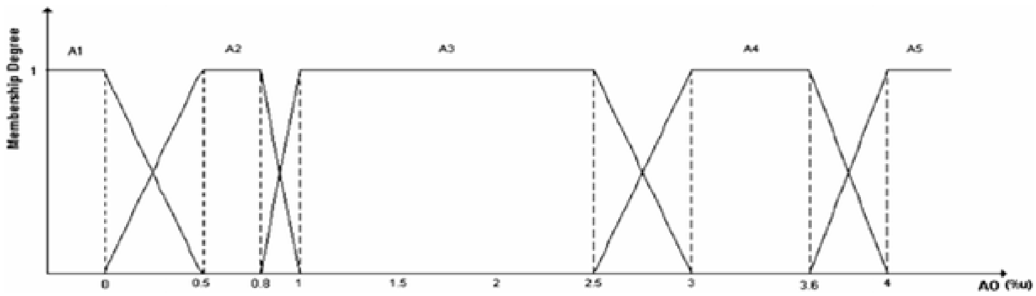


Figure 3.4: Fuzzy sets of the variable AO

	N1	N2	N3	N4	N5	N6	N7	N8
T1	A1	A1	A2	A2	A2	S	S	S
T2	A2	A2	A3	A3	A3	A4	A4	A4
T3	A2	A3	A3	A3	A3	A4	A4	A4
T4	A2	A3	A3	A3	A3	A4	A4	A4
T5	A2	A3	A3	A3	A3	A4	A4	A4
T6	A2	A3	A3	A3	A3	A4	A4	A4
T7	A2	A3	A3	A3	A3	A4	A4	A5
T8	S	A4	A4	A4	A4	A5	A5	A5
T9	S	A4	A4	A4	A4	A5	A5	A5
T10	S	A4	A4	A4	A4	A5	A5	A5
T11	S	A5	A5	A5	A5	A5	A5	A5
T12	S	A5	A5	A5	A5	A5	A5	A5

Figure 3.5: Decision matrix

Here the choices of operators and an example of fuzzy inference. This classic choice using the Zadeh operators and the Mamdani implication is called the Mamdani's fuzzy inference method.

Operator	Name
AND	min
OR	max
Implication	min
Aggregation	max
Defuzzification	COG

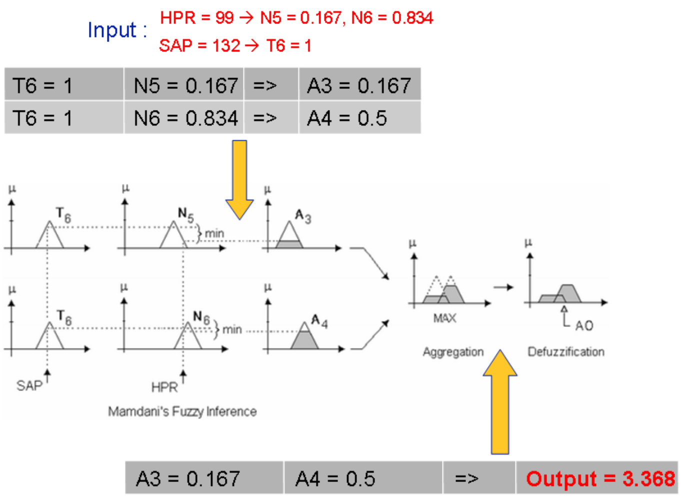


Figure 3.6: Example of inference

Now that the fuzzy system is established, let's compare it with the decisions taken by anesthesiologists:

												Total
Patient 1	SAP (mmHg)		164	161	192	156	172	161	152	157	154	
	HPR (p/m)		65	96	99	80	72	81	73	78	73	
	AO	Anesthetist	2	2.5	4	3	3.5	2.5	2.5	2.5	2.5	25
		Fuzzy Logic	3.25	3.29	3.88	2.18	3.27	3.27	1.83	2.38	1.83	25.2
	Distance %											+1
Patient 2	SAP (mmHg)		149	184	139	182	163	158	158			
	HPR (p/m)		88	94	98	74	78	90	87			
	AO	Anesthetist	2	4	2	5	4	2	2			21
		Fuzzy Logic	1.85	3.27	2.61	3.27	3.27	2.61	2.52			19.4
	Distance %											−7.6
Patient 3	SAP (mmHg)		110	130	130	110	130	120	120			
	HPR (p/m)		90	70	70	68	72	75	76			
	AO	Anesthetist	2.5	2	2	2	2	1.5	1			12
		Fuzzy Logic	1.83	1.83	1.83	1.85	1.83	1.84	1.85			11.1
	Distance %											−7.5
Patient 4	SAP (mmHg)		150	110	115	110	110					
	HPR (p/m)		96	74	107	95	90					
	AO	Anesthetist	3	2	2	2	2					11
		Fuzzy Logic	2.18	1.83	3.26	2.00	1.83					11.1
	Distance %											+1

Figure 3.7: Comparison between the decisions of a fuzzy system and those of a human

This table provides a comparison between doses of anesthetic products given by the anesthesiologist and those given by the fuzzy system every five minutes based on the SAP and HPR variables of the patient. Four operations of different durations are compared: the operation of patient 1 lasted 45 minutes, the patients 2 and 3 35 minutes, and the patient 4 stayed on operating table only 25 minutes.

The article and the previous table, from which it is drawn, point out that the sum of the doses of anesthetics at the end of the operation is substantially similar between the anesthesiologist and the fuzzy system: for example, for the first operation, the anesthesiologist gives 25 AO while the fuzzy system gives 25.2 AO, which is virtually the same. The article concludes that the fuzzy system is efficient enough to mimic the decisions of anesthesiologists.

However, comparing only the sum of AO given is not at all satisfactory. Indeed, it could very well happen that the first decision of the dose of anesthetics to provide to the patient is lethal, and subsequent doses are almost nil, which would give an amount comparable to the sum of a prescription made by an anesthesiologist.

The Pearson correlation coefficient allows us to have a better idea as to the correspondence between experimental and theoretical results. Of all the operations, the Pearson correlation coefficient is 0.627, which corresponds to a moderate approval, but often found in medical decisions when analyzing the difference between 2 physicians [Caroff, 2010].

A better indicator comparing experimental results with the results of the model is the square root of the mean square error (RMSE). In particular, the RMSE will allow us to compare several models.

Definition 11.

Let $\mathbf{A}_1 = \begin{bmatrix} x_{1,1} \\ x_{1,2} \\ \vdots \\ x_{1,n} \end{bmatrix}$ and $\mathbf{A}_2 = \begin{bmatrix} x_{2,1} \\ x_{2,2} \\ \vdots \\ x_{2,n} \end{bmatrix}$.

The square root of the mean square error is calculated by the formula:

$$\mathrm{RMSE}(\mathbf{A}_1, \mathbf{A}_2) = \sqrt{\mathrm{MSE}(\mathbf{A}_1, \mathbf{A}_2)} = \sqrt{\mathrm{E}((\mathbf{A}_1 - \mathbf{A}_2)^2)} = \sqrt{\frac{\sum_{i=1}^{n}(x_{1,i} - x_{2,i})^2}{n}}.$$

Of all operations, we find a RMSE equal to 0.6877. This will allow us to compare these results with the following experiment.

3.3 Experiment of video games rating

3.3.1 Presentation

Now let's look at the case where the human makes a decision based on criteria which he assess the value himself. We designed an experiment for this report in order to study this type of reasoning.

In short, the experiment asks seven subjects passionate about video games (*gamers*) to give scores to video games. Each of them will give a score to thirty video games and three days later, we will ask them to give a score to two criteria comprising all aspects of a video game. We then study the relationship between the notes of the two criteria with the overall score, to see if such an inference system based on fuzzy logic may have a similar relationship.

3.3.2 Protocol

Each subject will have the following profile: male, between 20 and 30, gamer. The experiment will be conducted in two sessions which last approximately 15 minutes each, separated by three days. Full details of the protocol are in Annex B.

During the first session, they will be asked to give a score to thirty games of their choice on a scale of 0 to 100, a high score meaning that the player enjoys the game

During the second session, they will be asked to give a score to always between 0 and 100 two criteria comprising all aspects of a video game. The two criteria:

- Criterion 1: technical aspects, including graphics, fluidity, quality of the interface, quality of the controls, online mode and the number of addons/extensions.
- Criterion 2: non-technical aspects, including the duration of the game, story, fun to play, interest in strategies, business community and the possibility of learning.

3.3.3 Results

Here is the graph of a subject and the graph for all subjects. A blue dot corresponds to the rating of a video game. Our data are in three dimensions:

1. Dimension 1: technical score
2. Dimension 2: non-technical score

3. Dimension 3: Overall score

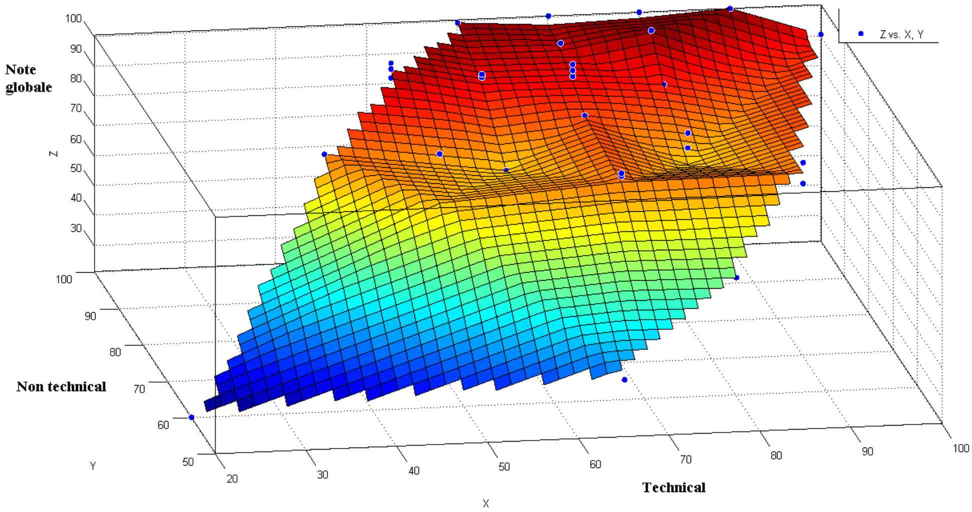


Figure 3.8: Surface generated from the scores of the subject 1 for the 2 criteria

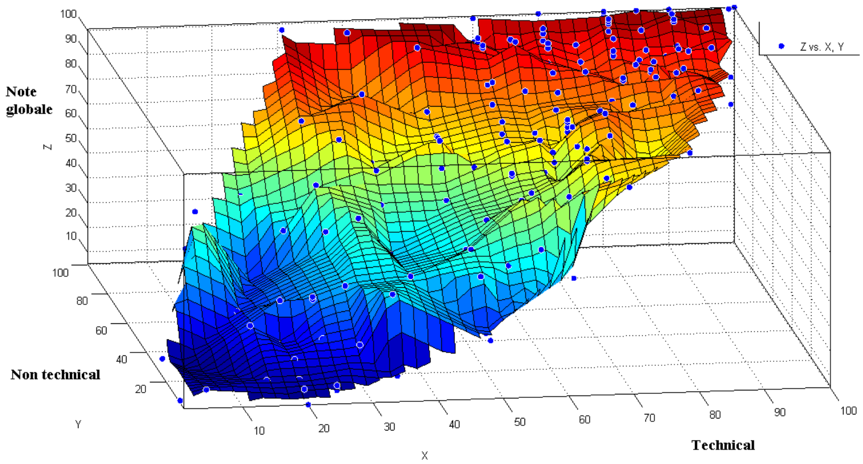


Figure 3.9: Surface generated from the scores of all subjects for the 2 criteria

We will compare the RMSE obtained with a polynomial prediction model with that obtained with a model based on a fuzzy system. The polynomial model with a RMSE equal to 7.91 when the degrees of X and Y is 1, and drops to 6.097 when X and Y have a degree equal to 5.

We must now develop a model based on a fuzzy system. Specifically, we need to define the fuzzy sets as well as the fuzzy rules of the decision matrix. The problem is that if we manually created the model, we would have no assurance of optimality and therefore the comparison of RMSE would not be necessarily relevant. In order to optimize as much as possible our fuzzy system, we will refine our parameters using neural networks. Such systems are called neuro-fuzzy systems.

3.3.4 Neuro-fuzzy systems

Neuro-fuzzy systems were introduced in the thesis of Jyh-Shing Roger Jang in 1992 under the name "Adaptative-Networks-based Fuzzy Inference Systems" (ANFIS) [Jangi, 1992]. They use the formalism of neural networks by expressing the structure of a fuzzy system in the form of a multilayer perceptron.

A multilayer perceptron (MLP) is a neural network without cycle. The input layer is given a vector network and the network returns a result vector in the output layer. Between these two layers, the elements of the input vector are weighted by the weights of the connections and mixed in the hidden neurons located in the hidden layer.

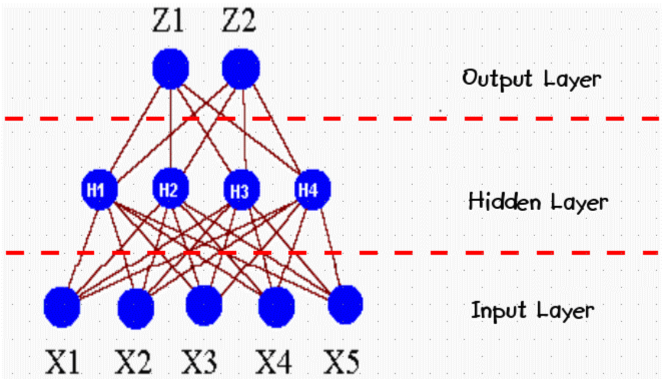


Figure 3.10: Example of a feedforward neural network

Several activation functions for the output layer are commonly used, such as linear, logistic or softmax. Similarly, there are several error backpropagation algorithms that optimize the learning of weights from the mistakes made between the values computed by the network and the actual values: Conjugate gradient optimization, Scaled Conjugate Gradient , Quasi-Newton optimization, and so on.

Here is the organization of our multilayer perceptron representing the neuro-fuzzy system which we will use to model the scoring system:

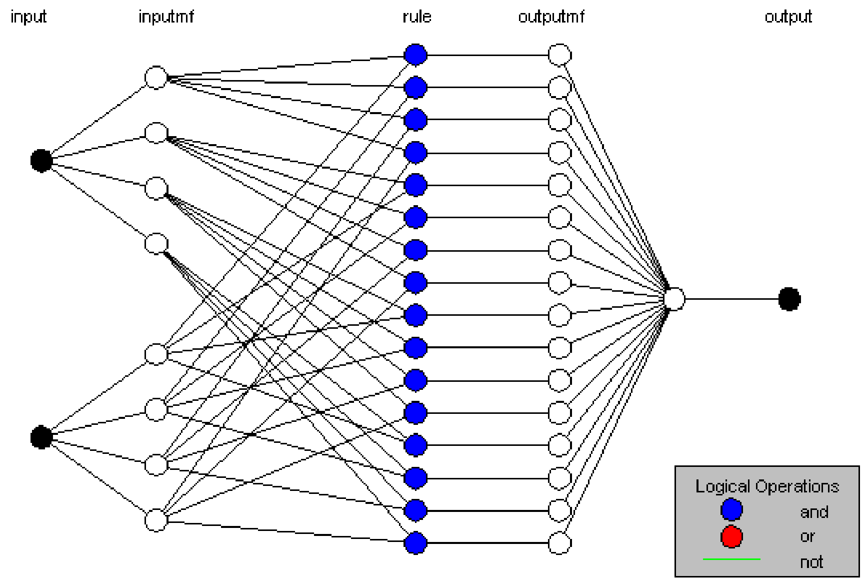


Figure 3.11: Structure of a neuro-fuzzy system

The field being new, we had to take into account several constraints in order to find a model for our video game scoring experiment. We had to fix before the learning phase:

- the number of input fuzzy sets: 4 per input,
- the shape of the membership function: Gaussian.

Here is the learning curve for our fuzzy system:

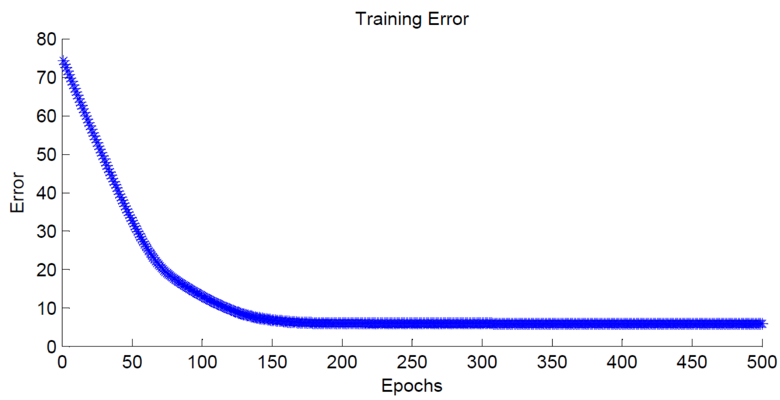


Figure 3.12: Learning curve of the neuro-fuzzy system

We obtain an RMSE of 5.9 after a few hundred steps of learning, far less than the RMSE of a polynomial of degree 1, and slightly lower than the RMSE of a polynomial of degree 5. This result is quite good, here is the decision surface obtained:

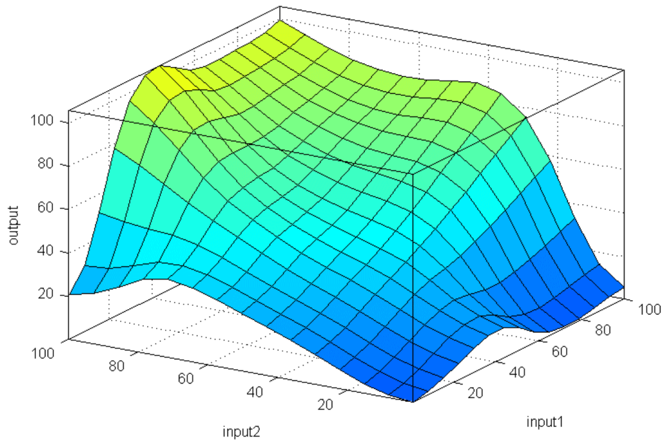


Figure 3.13: Decision surface of the neuro-fuzzy system

We notice that the decision surface of the fuzzy system we obtained has a form between a linear surface and a *classique* decision surface of a fuzzy system. The Pearson correlation coefficient between the two inputs is 0.786, meaning that subjects tended in the second stage of the experiment to be influenced by the technical non-technical notes that they had given before, or vice versa. However, even in these conditions which favor a polynomial model, the fuzzy model still has better results.

We see that the experimental data are close to the theoretical model, with the exception of two areas where extreme notes given to the technical side is very different from the note given to non-technical aspects, which is a case that never happens in practice because if the technical aspect of the game is really bad, it negatively impacts non-technical aspects, and vice versa if the non-technical aspects are too bad, the player stops playing right away before he can really assess the technical side.

Let's have a look at the decision matrix of our fuzzy system optimized by neural networks. As we have seen, we set to four the number of fuzzy sets for each of the two inputs:

- the set of fuzzy sets of input 1: $\{in_1mf_i, i \in [\![1,4]\!]\}$;
- the set of fuzzy sets of input 2: $\{in_2mf_i, i \in [\![1,4]\!]\}$;

- the set of fuzzy sets of output: $\{out_1mf_i, i \in [\![1, 4 \times 4]\!]\}$.

The set of fuzzy rules of our system is:
$\{If\ in_1mf_i\ and\ in_2mf_j\ then\ out_1mf_{4\times(i-1)+j}, (i,j) \in [\![1,4]\!]^2\}$ which appears cognitively plausible.

3.3.5 Comparison with the previous experiment

For the experiment with anesthesiologists, we created a neuro-fuzzy model to see if we could improve the system created by hand in the article. Here are the decisions of our fuzzy system with respect to the decisions of the anesthesiologist during the four operations for which we have the data:

Figure 3.14: Decision surface of the neuro-fuzzy system

Here is the summary of the RMSEs:

Experiment	Model	RMSE*	Standardized RMSE **
Anesthesiologist	Neuro-fuzzy system (by hand)	0,6877	13,754
Anesthesiologist	Neuro-fuzzy system	0,569	11,38
Game scoring	Polynomial model with degree 1	7,91	7,91
Game scoring	Polynomial model with degree 5	6,097	6,097
Game scoring	Neuro-fuzzy system	5,9	5,9

* RMSE between model and experimental data
** RMSE standardized (i.e. normalised) because the RMSE is influenced by the

scale. The output of the experiment with anesthesia has values between 0 and 5, whereas the output of the experiment with the scores has values between 0 and 100. Therefore, we normalize the RMSE by multiplying by $\frac{100}{5} = 20$.

We see that for the experiment with the anesthesiologists, our fuzzy system has a slightly better RMSE than that proposed in the article. However, it should have more data because the article gives the data of only 4 operations, which gives us only 28 individuals in the statistical series, which is very inadequate to effectively optimize a fuzzy system with neural networks. Here are two decision surfaces. The first surface is the one that allowed us to obtain a RMSE equal to 0.569: we see that the form is quite correct except for high values of input 2, reflecting the lack of data there. The second is an example of over-learning increased by an excessive number of fuzzy sets, which has a RMSE of 0.212.

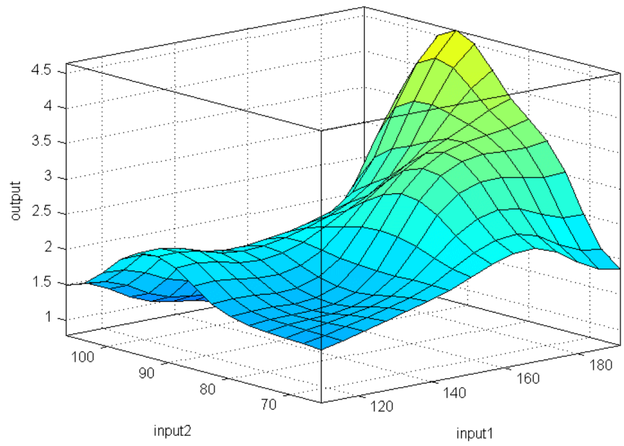


Figure 3.15: Decision surface of the neuro-fuzzy system

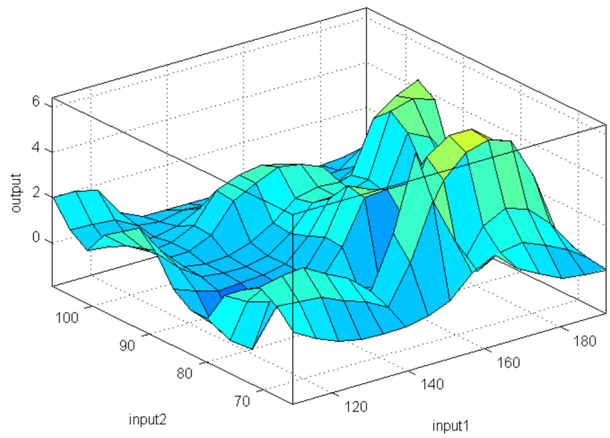


Figure 3.16: Decision surface of a over-learned neuro-fuzzy system

To compare the RMSE between the two experiments, after normalization we see that our fuzzy system for the game experiment has a RMSE (5.9) well below the fuzzy system defined by hand for anesthesiologists (13.754). This is not surprising as the number of data in the game experiment is about 10 times larger (267 against 28), which allowed the neural network of well optimized fuzzy system while making almost impossible to over-learning, because the number of fuzzy sets of inputs is limited.
It is yet difficult to compare the RMSE between the two experiments because in the first one the two input variables are almost uncorrelated, while in the second one they are more correlated.

3.4 Conclusions

In conclusion, fuzzy systems seem to be able to generate quite similar results from the decisions of human reasoning. Clearly, they are better than linear systems. In addition, fuzzy systems used in these experiments, particularly for the second, are very simple: two inputs and three or four fuzzy sets for each.

Fuzzy logic makes it possible to imitate in these examples some of the reasoning of the human subject, if we first define the parameters of fuzzy inference system (membership function, choice of the rule of implication and so on).

Chapter 4

Conclusion

We saw in the first part that fuzzy logic stems from the need to formalize inaccuracies. Despite its simple rules, it is able to mathematically model inference systems much more complex than can classical logic and linear models. Its decision matrix that brings together the fuzzy rules is similar in both form (structure of type *If X and Y then Z*) and content (the concept of linguistic variables) to the type of rules orally expressed by humans.

Besides, fuzzy logic can explain many experiments that had undermined traditional models of human reasoning in the 20th century. We showed how the non-additivity of probability judgments can be expressed in a fuzzy system. We then confronted fuzzy logic with some paradoxes of classical logic when it tries to model human reasoning: the sorites paradox is typically the kind of threshold problem that fuzzy logic reduces and the paradox of entailment does not pose a problem in fuzzy logic. It would be interesting to further explore Hempel's paradox and especially how we could express it in a neuro-fuzzy system. Similarly, Wason selection task would require further analysis, this time by focusing on fuzzy modus ponens and modus tollens. Beyond these paradoxes and non-additivity, fuzzy logic, based on the concept of linguistic variables, is essentially similar to natural language.

Thus fuzzy logic appears as a powerful theoretical framework for studying human reasoning. Surprisingly, we found only one study comparing the decisions made by humans with that of a fuzzy system, whose purpose was essentially to design a system of decision support for medical personnel, not analyze human reasoning as such. We conducted our own experiment and investigated whether a fuzzy system could mimic the results observed in humans. For this purpose, we used a technique for optimizing fuzzy system using neural networks (neuro-fuzzy), through which we obtained good results, although the correlation between the two criteria for entry is high: a fuzzy system gives results closer to experimental values than those obtained by a polynomial system. This result reinforces the hypothesis that fuzzy logic can be used to explain decisions from human reasoning.

However, we emphasize on the fact on these experiments show at best that fuzzy systems can **imitate** human reasoning: only neuroscience could show that this way of reasoning really exists in the human brain. On this subject, some research articles of neuroscience such as [A., 2005] or [Feng and Capranica, 1978] in 2008 commented by David Olmsted on the website http://www.neurocomputing.org suggest that it is plausible that the concepts of fuzzy logic control means are "biologically compatible".

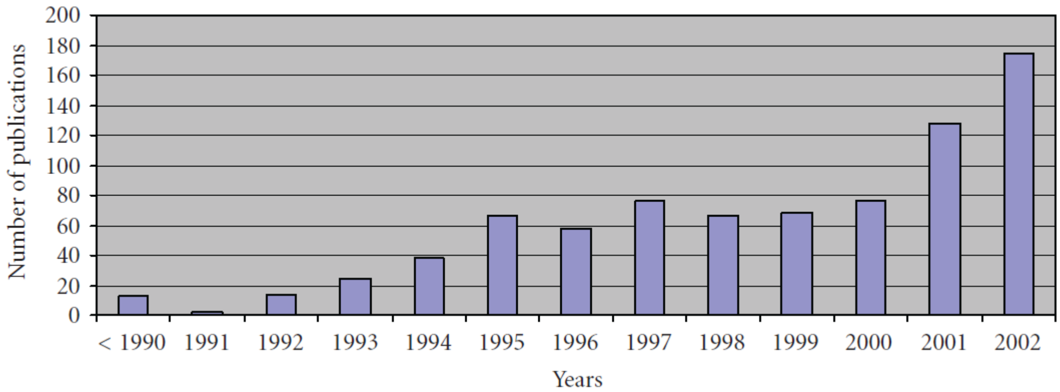


Figure 4.1: Number of publications per year indexed in MEDLINE using fuzzy logic. Source: [Torres A., 2005]

As shown in Figure 4.1, interest in fuzzy logic is growing: having developed first in the industrial world, this theory, which ultimately is highly empirical, interests other fields such as medicine (decision support) and engineering decision (fuzzy databases). The psychology of human reasoning, however, does not seem to have been extensively studied this area.

This is where lies the paradox: the computer side of artificial intelligence has been really interested in fuzzy logic as it allows to bind to the psychological side. Yet the latter does not seem to be so interested in fuzzy logic so far.

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Appendices

Appendix A

Experiment with the anaesthesiologists

Here is the table of correlation calculated with SPSS:

Correlations

		VAR00013	VAR00014
VAR00013	Pearson Correlation	1	,627**
	Sig. (2-tailed)		,000
	N	28	28
VAR00014	Pearson Correlation	,627**	1
	Sig. (2-tailed)	,000	
	N	28	28

**. Correlation is significant at the 0.01 level (2-tailed).

Figure A.1: Correlation matrix between the decisions of the anesthesiologist and those of the fuzzy system

The script to compute the correlation matrix is:

```
------------------- SCRIPT SPSS -------------------
DATASET ACTIVATE DataSet0.
CORRELATIONS
/VARIABLES=VAR00013 VAR00014
/PRINT=BOTH TWOTAIL NOSIG
/MISSING=PAIRWISE.
------------------------------------------------------
```

Appendix B

Game scoring experiment

Here are the details of the game scoring experiment. Some results as well as analysis scripts will be placed here as well.

During the first session, subjects are asked to score thirty games of their choice on a scale of 0 to 100, a high rating meaning that the player enjoys the game, on the website Wiki4Games (http://www.wiki4games.com). This website is based on the *MediaWiki* engine, and is used by all projects of the Wikimedia Foundation, including encyclopedic site *Wikipédia*. To record scores, they will use the extension *W4G Rating Bar*, which is a bar allowing each registered user on the site to give a score. Wiki4Games site is administered by the author of this document, and the extension *W4G Rating Bar* was developed by himself for the motor MediaWiki (http://www.mediawiki.org/wiki/Extension:W4G_Rating_Bar). This will allow total control of the scoring process of the experiment.

During the second session, they are asked to note two criteria comprising all aspects of a video game on an Excel spreadsheet. To analyze the results, we used MATLAB and SPSS.

In particular, we used two MATLAB tools:

- The *Surface Fitting Tool* allows us to generate a surface from 3D points of the experimental data.
- The *Fuzzy Loggic Toolbox* (http://www.mathworks.com/products/fuzzylogic/ allows us to study the results we get from an inference system based on fuzzy logic.

To establish the charts in the scores of the subjects, we store the stored the data in a .mat file and we will use the following script: *notesDavid.mat*:

```
------------------- SCRIPT MATLAB -------------------
% load data
load('notesDavid.mat', 'data')
selectedNotes = data;
X = selectedNotes(:, 1); % technical aspects
Y = selectedNotes(:, 2); % non technical aspects
Z = selectedNotes(:, 3); % global note

% draw figure
figure
scatter3(X, Y, Z, 5, 'filled')

% call the Surface Fitting Tool
sftool(X, Y, Z)
------------------------------------------------------
```

The following pages show the results for subjects 1 and 2.

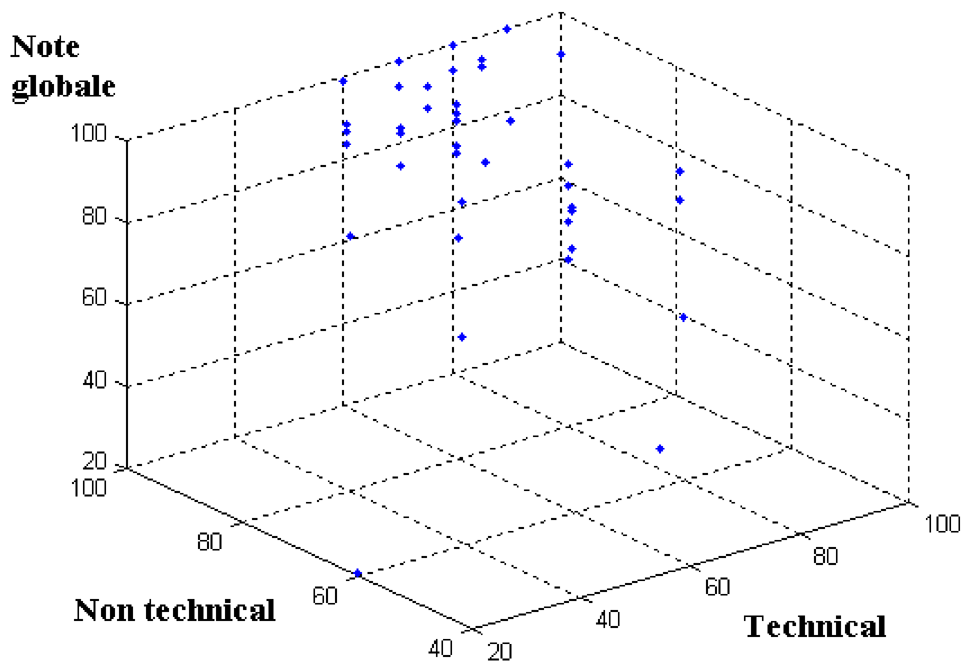

Figure B.1: Overall scores of the subject 1 for the 2 criteria

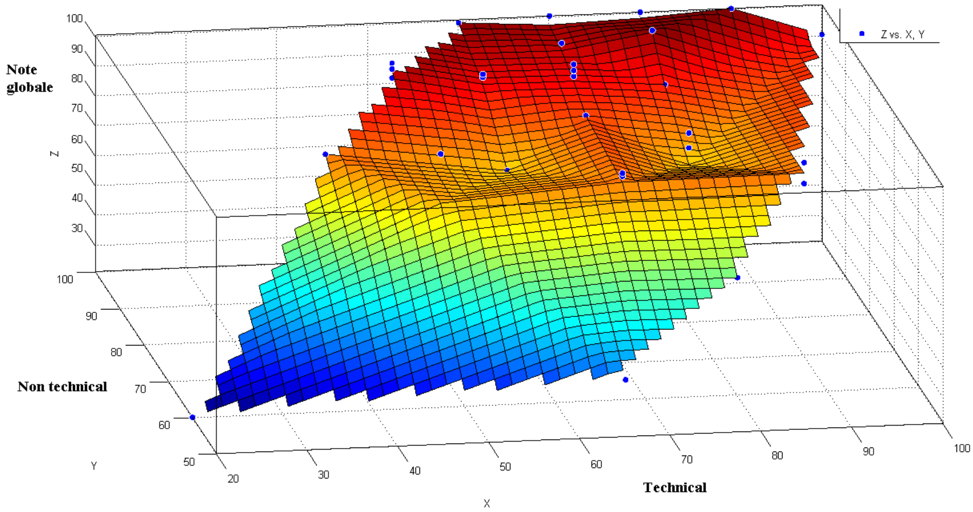

Figure B.2: Surface generated from the overall scores of the subject 1 for the 2 criteria

The RMSE were calculated in MATLAB using the function errperf available here: http://www.mathworks.com/matlabcentral/fileexchange/15130-error-related-performance-metrics.

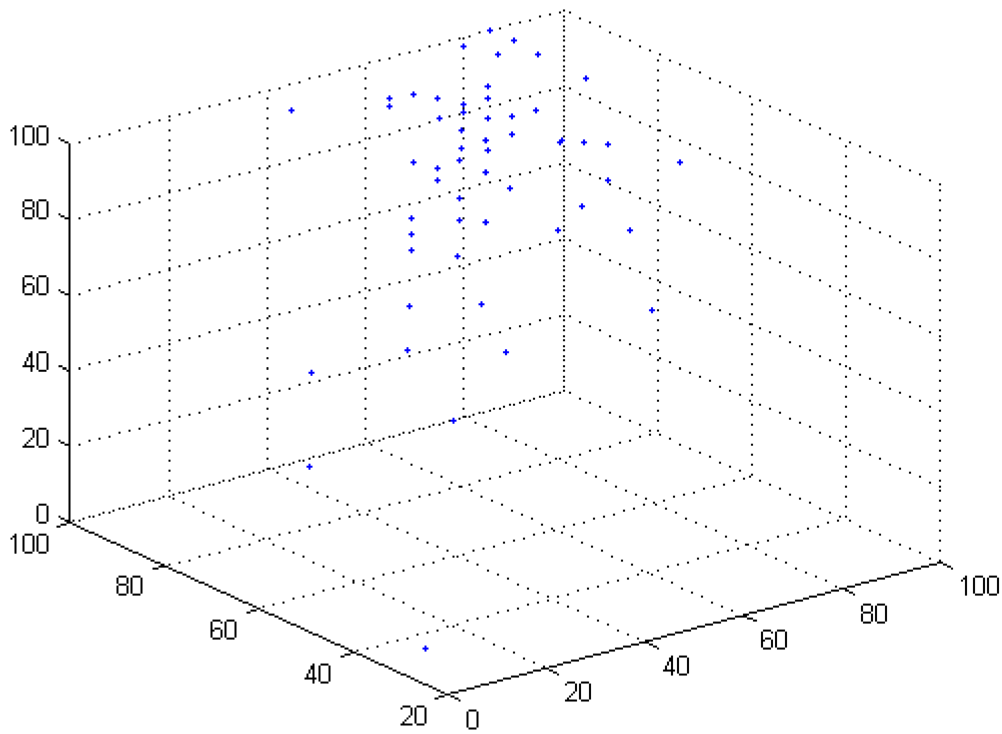

Figure B.3: Overall scores of subject 2 for the 2 criteria

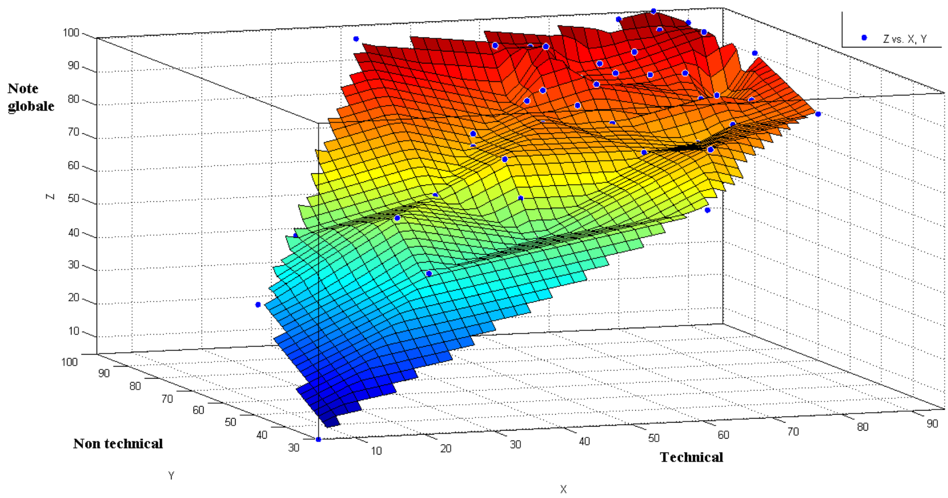

Figure B.4: Surface generated from overall scores of subject 2 for the 2 criteria